No. of Printed Pages : 2

PG-271

10141

IV Semester M.Sc. (CBCS) Examination, July - 2019

MATHEMATICS

M403T(B)-Y2K17 M402TB-Y2K14 : Special Functions

Time : 3 Hours | Max. Marks : 70

Instructions : *(1) Answer **any five** full questions.*
*(2) Each question carries **equal** marks.*

1. (a) Define hypergeometric series. Derive convergence conditions for ordinary hypergeometric series. **7+7**

   (b) Derive the integral representation for ordinary hypergeometric series ${}_2F_1(a, b; c ; z)$.

2. (a) For $Re(c-a-b)>0$, prove that **7+7**

$${}_2F_1(a, b; c; 1) = \frac{\Gamma(c)\,\Gamma(c-a-b)}{\Gamma(c-a)\,\Gamma(c-b)}$$

   Hence deduce chu-Vandermonde summation formula.

   (b) Prove that ${}_2F_1(a, b ; c ; z) = (1-z)^{c-a-b}\ {}_2F_1(c-a, c-b ; c ; z)$, by deriving pfaff-kummer transformation formula.

3. (a) State and prove q-binomial theorem. **7+3+4**

   (b) Prove the following :

   (i) $$\frac{1}{(z ; q)_\infty} = \sum_{n=0}^{\infty} \frac{z^n}{(q; q)_n}$$

   (ii) $$(-z ; q)_\infty = \sum_{n=0}^{\infty} \frac{q^{n(n-1)/2}}{(q; q)_n} z^n$$

   (c) Derive Heine's formula.

4. (a) State Jacobi-triple product identity. Hence, deduce Jacobi identity. **6+8**

   (b) For $\left|\frac{b}{a}\right| < |z| < 1$ and $|q| < 1$, then prove that

$$\sum_{n=-\infty}^{\infty} \frac{(a)_n}{(b)_n} z^n = \frac{(az ; q)_\infty \left(\frac{q}{az} ; q\right)_\infty (q ; q)_\infty \left(\frac{b}{a} ; q\right)_\infty}{(z ; q)_\infty \left(\frac{b}{az} ; q\right)_\infty (b ; q)_\infty \left(\frac{q}{a} ; q\right)_\infty}$$

P.T.O.

5. State and prove quintuple product identity. 14

6. Prove the following : 3+3+5+3

(i) $\log Q(q) = 2 \sum_{k=1}^{\infty} \frac{q^{2k-1}}{(2k-1)\left(1+q^{2k-1}\right)}$

(ii) $Q(q)+Q(-q)=2Q(q^4)$

(iii) $f(a, b)\, f(c, d)+f(-a, -b)\, f(-c, -d)=2f(ac, bd)\, f(ad, bc)$

(iv) $f^2(a, b)+f^2(-a, -b)=2f(a^2, b^2)Q(ab)$

7. (a) Define partition function p(n). Prove that the number of partitions of n with atmost m parts equals the number of partitions of n in which no parts exceeds m. 7+7

(b) Prove the following :

(i) $(1+x)(1+x^3)(1+x^5) \ldots = \sum_{n=0}^{\infty} \frac{x^{n^2}}{(x^2 ; x^2)_n}$

(ii) $(1+x^2)(1+x^4)(1+x^6) \ldots = \sum_{n=0}^{\infty} \frac{x^{n^2+n}}{(x^2 ; x^2)_n}$

8. (a) State and prove Euler's pentagonal number theorem. 8+6

(b) Prove that $p(5m+4)\equiv 0 \pmod 5$.

- o 0 o -